BENHA UNIVERSITY
FACULTY OF ENGINEERING AT SHOUBRA

ECE-312
Electronic Circuits (A)

Lecture # 8
BJT High Frequency Response

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Agenda

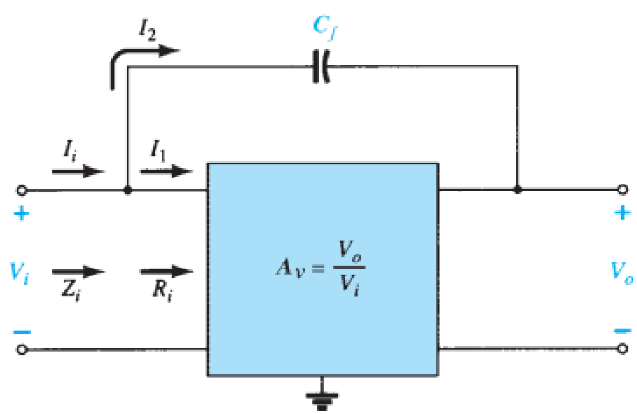
- Miller Effect Capacitance
- High Frequency Response – BJT Amplifier
- Multistage Frequency Effects
- Square-Wave Testing

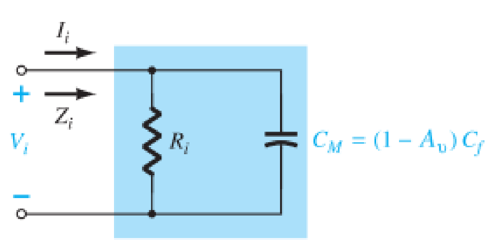
MILLER EFFECT CAPACITANCE

Miller input capacitance

$$\boxed{C_{M_i} = (1 - A_v)C_f}$$

- In the high-frequency region, the capacitive elements of importance are the interelectrode (between-terminals) capacitances internal to the active device and the wiring capacitance between leads of the network.
- For any inverting amplifier, the input capacitance will be increased by a Miller effect capacitance sensitive to the gain of the amplifier and the interelectrode (parasitic) capacitance between the input and output terminals of the active device.





Applying Kirchhoff's current law gives

$$I_i = I_1 + I_2$$

Using Ohm's law yields

$$I_i = \frac{V_i}{Z_i}, \quad I_1 = \frac{V_i}{R_i}$$

and

$$I_2 = \frac{V_i - V_o}{X_{C_f}} = \frac{V_i - A_v V_i}{X_{C_f}} = \frac{(1 - A_v)V_i}{X_{C_f}}$$

Substituting, we obtain

$$\frac{V_i}{Z_i} = \frac{V_i}{R_i} + \frac{(1 - A_v)V_i}{X_{C_f}}$$

and

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_f}/(1 - A_v)}$$

but

$$\frac{X_{C_f}}{1 - A_v} = \frac{1}{\omega\underbrace{(1 - A_v)C_f}_{C_M}} = X_{C_M}$$

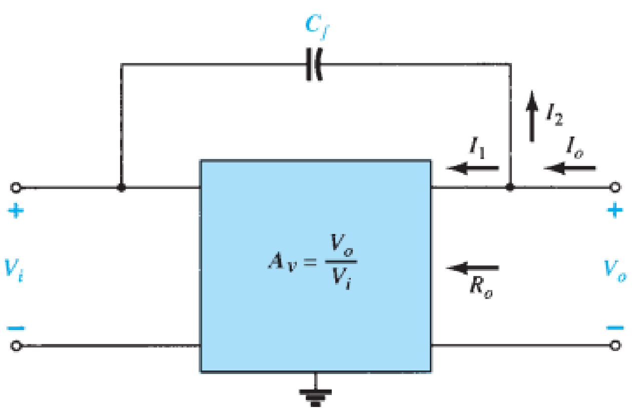
and

$$\frac{1}{Z_i} = \frac{1}{R_i} + \frac{1}{X_{C_M}}$$

Miller output capacitance

$$C_{M_o} = \left(1 - \frac{1}{A_v}\right) C_f$$

- A positive value for A_v would result in a negative capacitance (for Av > 1).
- For noninverting amplifiers such as the common-base and emitter-follower configurations, the Miller effect capacitance is not a contributing concern for high-frequency applications.
- The Miller effect will also increase the level of output capacitance, which must also be considered when the high-frequency cutoff is determined.



$$C_{M_o} = \left(1 - \frac{1}{A_v}\right) C_f$$

$$C_{M_o} \cong C_f \quad |A_v| \gg 1$$

$$I_o = I_1 + I_2$$

$$I_1 = \frac{V_o}{R_o} \qquad \text{and} \qquad I_2 = \frac{V_o - V_i}{X_{C_f}}$$

The resistance R_o is usually sufficiently large to permit ignoring the first term of the equation compared to the second term and assuming that

$$I_o \cong \frac{V_o - V_i}{X_{C_f}}$$

Substituting $V_i = V_o/A_v$ from $A_v = V_o/V_i$ results in

$$I_o = \frac{V_o - V_o/A_v}{X_{C_f}} = \frac{V_o(1 - 1/A_v)}{X_{C_f}}$$

and

$$\frac{I_o}{V_o} = \frac{1 - 1/A_v}{X_{C_f}}$$

or

$$\frac{V_o}{I_o} = \frac{X_{C_f}}{1 - 1/A_v} = \frac{1}{\omega C_f(1 - 1/A_v)} = \frac{1}{\omega C_{M_o}}$$

HIGH FREQUENCY RESPONSE – BJT AMPLIFIER

High Frequency Response

- At the high-frequency end, there are two factors that define the 3-dB cutoff point:
  1. the network capacitance (parasitic and introduced)
  2. the frequency dependence of h_{fe} (β).

- For RC circuit:

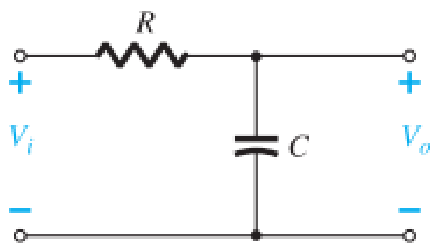
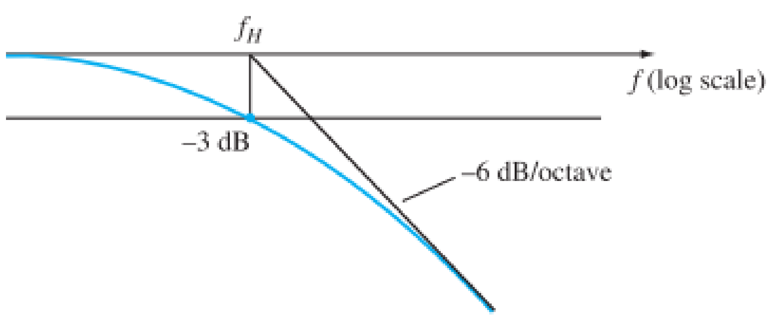


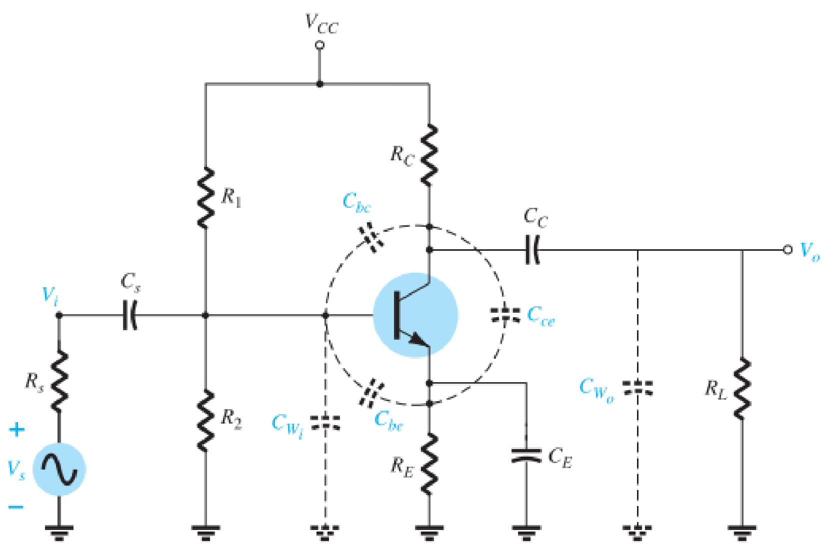
FIG. 9.45
RC combination that will define a high-cutoff frequency.



$$A_v = \frac{1}{1 + j(f/f_H)}$$

1. Network Parameters :

- At high frequencies, the various parasitic capacitances (C_{be}, C_{bc}, C_{ce}) of the transistor are included with the wiring capacitances (C_{Wi}, C_{Wo}).



$$f_{H_i} = \frac{1}{2\pi R_{\mathrm{Th}_i} C_i}$$

$$R_{\mathrm{Th}_i} = R_s \| R_1 \| R_2 \| \beta r_e$$

$$C_i = C_{W_i} + C_{be} + C_{M_i} = C_{W_i} + C_{be} + (1 - A_v)C_{bc}$$

$$f_{H_o} = \frac{1}{2\pi R_{\mathrm{Th}_o} C_o}$$

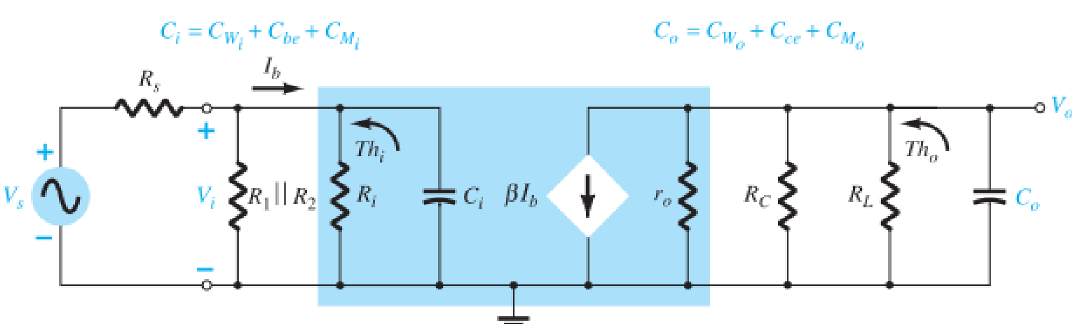
$$R_{\mathrm{Th}_o} = R_C \| R_L \| r_o$$

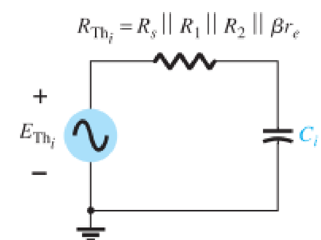
$$C_o = C_{W_o} + C_{ce} + C_{M_o}$$

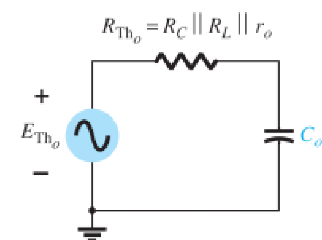
$$C_o = C_{W_o} + C_{ce} + (1 - 1/A_v)C_{bc}$$

$$1 \gg 1/A_v$$

$$C_o \cong C_{W_o} + C_{ce} + C_{bc}$$







2. h_{fe} (or β) Variation

- The variation of h_{fe} (or β) with frequency approaches the following relationship:

$$h_{fe} = \frac{h_{fe_{\text{mid}}}}{1 + j(f/f_\beta)}$$

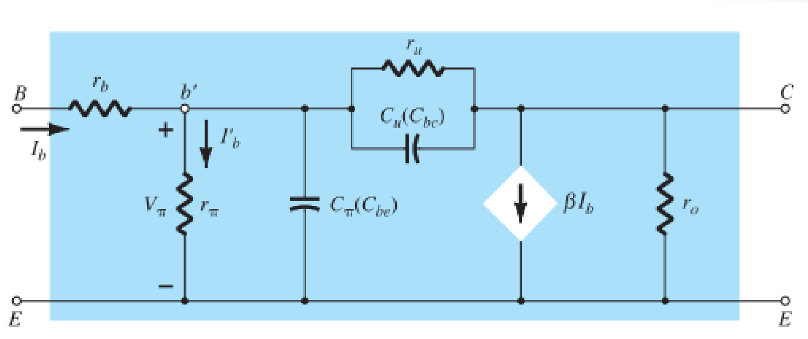
- The quantity, f_β, is determined by a set of parameters employed in the hybrid π model

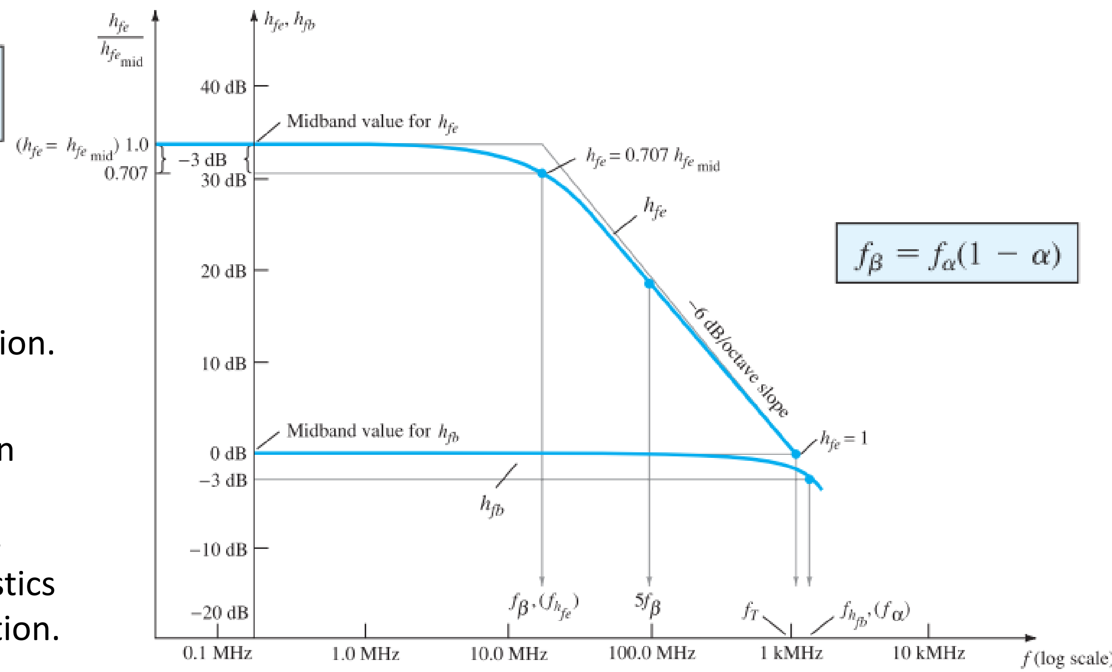
$$f_\beta(\text{often appearing as } f_{h_{fe}}) = \frac{1}{2\pi r_\pi(C_\pi + C_u)}$$

$$f_\beta = \frac{1}{h_{fe_{\text{mid}}}} \frac{1}{2\pi r_e(C_\pi + C_u)}$$

- f_β is a function of the bias configuration.

- the small change in h_{fb} for the chosen frequency range, revealing that the common-base configuration displays improved high-frequency characteristics over the common-emitter configuration.





$$f_\beta = f_\alpha(1 - \alpha)$$

FIG. 9.51

h_{fe} and h_{fb} versus frequency in the high-frequency region.

Gain-Bandwidth Product

- There is a Figure of Merit applied to amplifiers called the Gain-Bandwidth Product (GBP) that is commonly used to initiate the design process of an amplifier.
- It provides important information about the relationship between the gain of the amplifier and the expected operating frequency range.

$$\boxed{GBP = A_{v_{mid}} BW}$$

$$BW = f_H - f_L \cong f_H$$

$$\boxed{f_T = A_{v_{mid}} f_H} \quad \text{(Hz)}$$

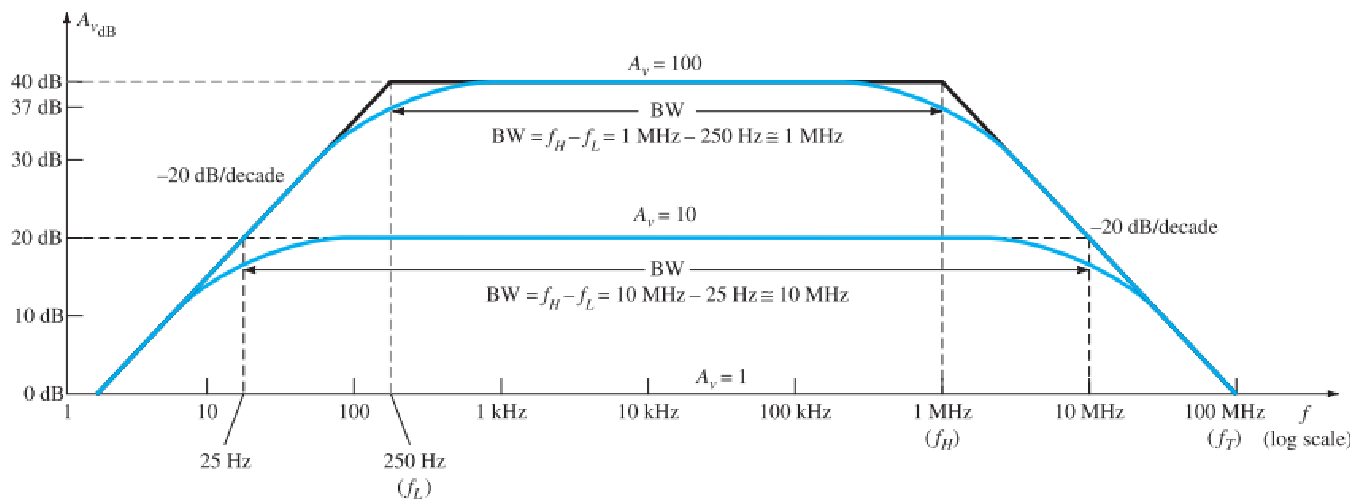


FIG. 9.53
Finding the bandwidth at two different gain levels.

- at any level of gain the product of the two remains a constant.
- the frequency f_T is called the unity-gain frequency and is always equal to the product of the midband gain of an amplifier and the bandwidth at any level of gain.

- For transistors:

$$f_T = h_{fe_{mid}} \frac{1}{2\pi h_{fe_{mid}} r_e (C_\pi + C_u)}$$

$$\boxed{f_T = h_{fe_{mid}} f_\beta} \quad \text{(Hz)}$$

$$\boxed{f_T \cong \frac{1}{2\pi r_e (C_\pi + C_u)}}$$

Example

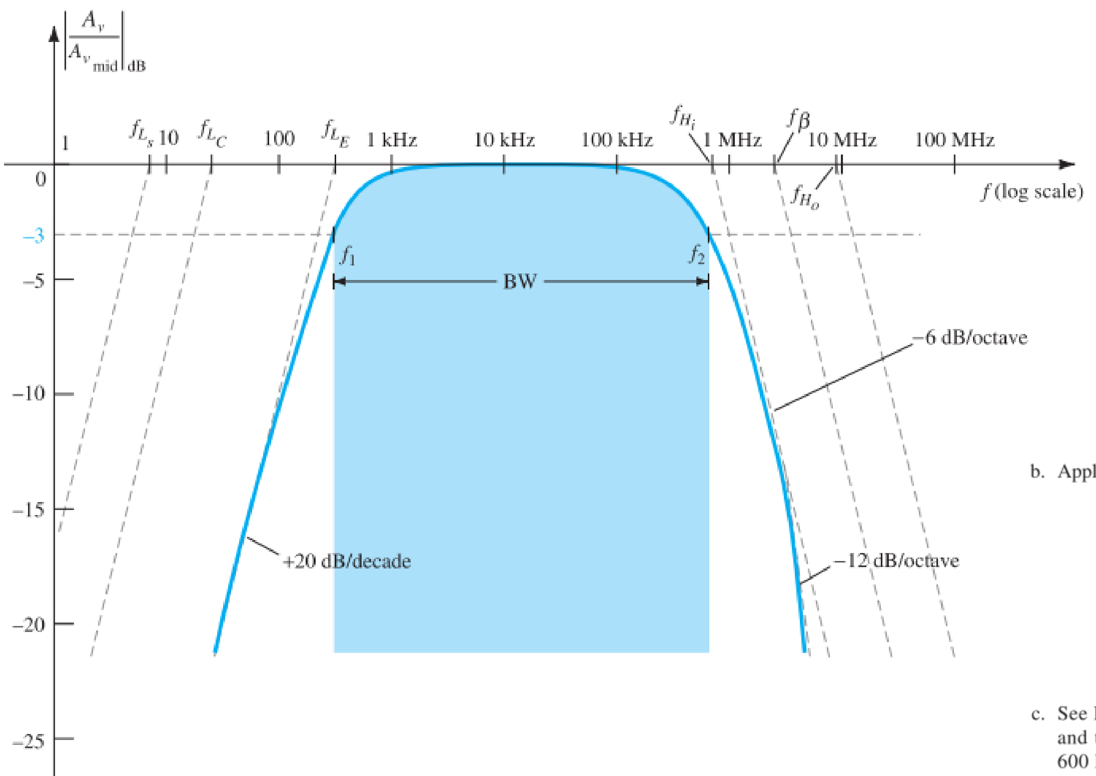
EXAMPLE 9.14 Use the network of Fig. 9.47 with the same parameters as in Example 9.12, that is,

$$R_s = 1\text{ k}\Omega, R_1 = 40\text{ k}\Omega, R_2 = 10\text{ k}\Omega, R_E = 2\text{ k}\Omega, R_C = 4\text{ k}\Omega, R_L = 2.2\text{ k}\Omega$$
$$C_s = 10\ \mu\text{F}, C_C = 1\ \mu\text{F}, C_E = 20\ \mu\text{F}$$
$$h_{fe} = 100, r_o = \infty\ \Omega, V_{CC} = 20\text{ V}$$

with the addition of

$$C_\pi(C_{be}) = 36\text{ pF}, C_u(C_{bc}) = 4\text{ pF}, C_{ce} = 1\text{ pF}, C_{W_i} = 6\text{ pF}, C_{W_o} = 8\text{ pF}$$

a. Determine f_{H_i} and f_{H_o}.
b. Find f_β and f_T.
c. Sketch the frequency response for the low- and high-frequency regions using the results of Example 9.12 and the results of parts (a) and (b).



Solution:

a. From Example 9.12:

$$\beta r_e = 1.576\text{ k}\Omega, \quad A_{v_{mid}}(\text{amplifier—not including effects of } R_s) = -90$$

and $$R_{Th_i} = R_s \| R_1 \| R_2 \| \beta r_e = 1\text{ k}\Omega \| 40\text{ k}\Omega \| 10\text{ k}\Omega \| 1.576\text{ k}\Omega \cong 0.57\text{ k}\Omega$$

with $$C_i = C_{W_i} + C_{be} + (1 - A_v)C_{bc} = 6\text{ pF} + 36\text{ pF} + [1 - (-90)]4\text{ pF} = 406\text{ pF}$$

$$f_{H_i} = \frac{1}{2\pi R_{Th_i} C_i} = \frac{1}{2\pi(0.57\text{ k}\Omega)(406\text{ pF})} = \mathbf{687.73\text{ kHz}}$$

$$R_{Th_o} = R_C \| R_L = 4\text{ k}\Omega \| 2.2\text{ k}\Omega = 1.419\text{ k}\Omega$$

$$C_o = C_{W_o} + C_{ce} + C_{M_o} = 8\text{ pF} + 1\text{ pF} + \left(1 - \frac{1}{-90}\right)4\text{ pF} = 13.04\text{ pF}$$

$$f_{H_o} = \frac{1}{2\pi R_{Th_o} C_o} = \frac{1}{2\pi(1.419\text{ k}\Omega)(13.04\text{ pF})} = \mathbf{8.6\text{ MHz}}$$

b. Applying Eq. (9.63) gives

$$f_\beta = \frac{1}{2\pi h_{fe_{mid}} r_e (C_{be} + C_{bc})} = \frac{1}{2\pi(100)(15.76\ \Omega)(36\text{ pF} + 4\text{ pF})} = \frac{1}{2\pi(100)(15.76\ \Omega)(40\text{ pF})} = \mathbf{2.52\text{ MHz}}$$

$$f_T = h_{fe_{mid}} f_\beta = (100)(2.52\text{ MHz}) = \mathbf{252\text{ MHz}}$$

c. See Fig. 9.54. The corner frequency f_{H_i} will determine the high cutoff frequency and the bandwidth of the amplifier. The upper cutoff frequency is very close to 600 kHz.

MULTISTAGE FREQUENCY EFFECTS

Multistage Frequency Effects

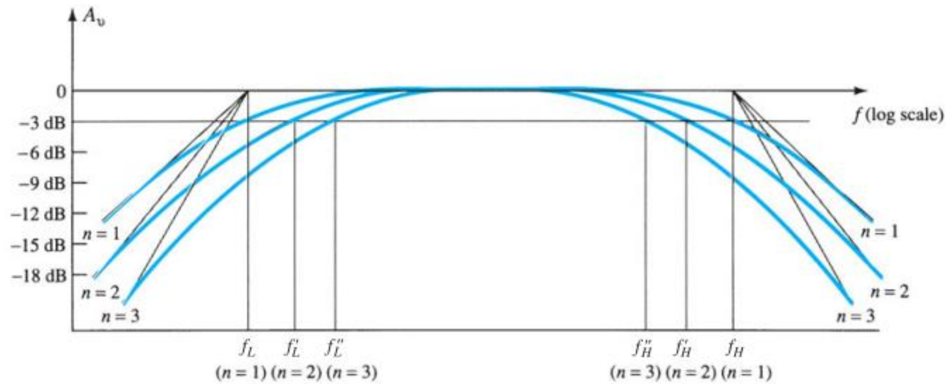


FIG. 9.58
Effect of an increased number of stages on the cutoff frequencies and the bandwidth.

$$A_{v_{\text{low, (overall)}}} = A_{v_{1_{\text{low}}}} A_{v_{2_{\text{low}}}} A_{v_{3_{\text{low}}}} \cdots A_{v_{n_{\text{low}}}}$$

but because all stages are identical, $A_{v_{1_{\text{low}}}} = A_{v_{2_{\text{low}}}} =$ etc., and

$$A_{v_{\text{low, (overall)}}} = (A_{v_{1_{\text{low}}}})^n$$

or
$$\frac{A_{v_{\text{low}}}}{A_{v_{\text{mid}}}}(\text{overall}) = \left(\frac{A_{v_{\text{low}}}}{A_{v_{\text{mid}}}}\right)^n = \frac{1}{(1 - jf_L/f)^n}$$

Setting the magnitude of this result equal to $1/\sqrt{2}$(-3 dB level) results in

$$\frac{1}{\sqrt{[1 + (f_L/f'_L)^2]^n}} = \frac{1}{\sqrt{2}}$$

or
$$\left\{\left[1 + \left(\frac{f_L}{f'_L}\right)^{2}\right]^{1/2}\right\}^n = \left\{\left[1 + \left(\frac{f_L}{f'_L}\right)^2\right]^n\right\}^{1/2} = (2)^{1/2}$$

so that
$$\left[1 + \left(\frac{f_L}{f'_L}\right)^2\right]^n = 2$$

and
$$1 + \left(\frac{f_L}{f'_L}\right)^2 = 2^{1/n}$$

with the result that
$$\boxed{f'_L = \frac{f_L}{\sqrt{2^{1/n} - 1}}}$$

In a similar manner, it can be shown that for the high-frequency region,

$$\boxed{f'_H = (\sqrt{2^{1/n} - 1})f_H}$$

n	$\sqrt{2^{1/n} - 1}$
2	0.64
3	0.51
4	0.43
5	0.39

SQUARE-WAVE TESTING

Square-Wave Testing

- A sense for the frequency response of an amplifier can be determined experimentally by applying a square-wave signal to the amplifier and noting the output response.

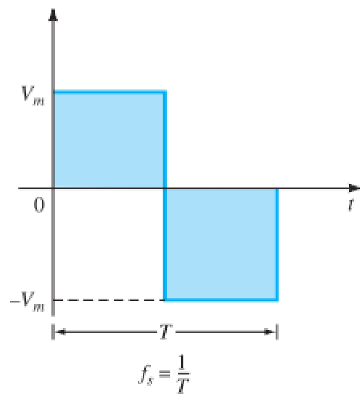


FIG. 9.59
Square wave

The Fourier series expansion for the square wave of Fig. 9.59 is

$$v = \frac{4}{\pi} V_m \left(\underbrace{\sin 2\pi f_s t}_{\text{fundamental}} + \underbrace{\frac{1}{3} \sin 2\pi(3f_s)t}_{\text{third harmonic}} + \underbrace{\frac{1}{5} \sin 2\pi(5f_s)t}_{\text{fifth harmonic}} + \underbrace{\frac{1}{7} \sin 2\pi(7f_s)t}_{\text{seventh harmonic}} + \underbrace{\frac{1}{9} \sin 2\pi(9f_s)t}_{\text{ninth harmonic}} + \cdots + \underbrace{\frac{1}{n} \sin 2\pi(nf_s)t}_{n\text{th harmonic}} \right)$$

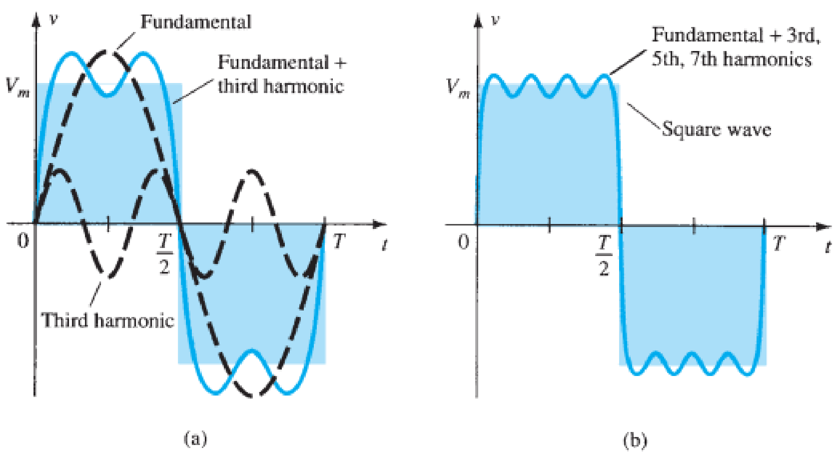


FIG. 9.60
Harmonic content of a square wave.

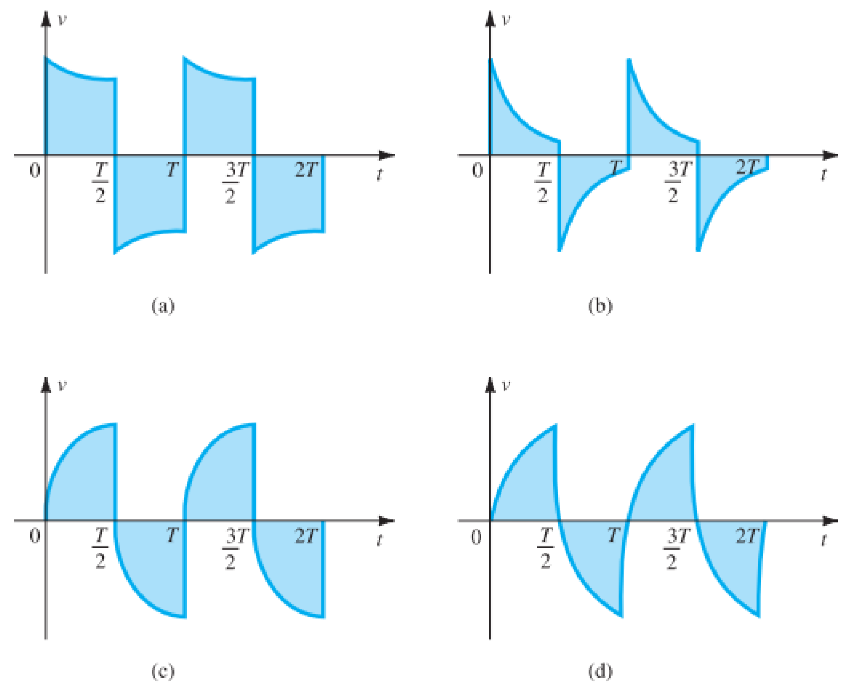


FIG. 9.61
(a) Poor low-frequency response; (b) very poor low-frequency response; (c) poor high-frequency response; (d) very poor high-frequency response.

Square-Wave Testing..

the BW of the amplifier:

$$\boxed{BW \cong f_{H_i} = \frac{0.35}{t_r}}$$

$$\boxed{\%\ \text{tilt} = P\% = \frac{V - V'}{V} \times 100\%}$$

$$\boxed{\text{tilt} = P = \frac{V - V'}{V}}$$

The low-cutoff frequency is then determined from

$$\boxed{f_{L_o} = \frac{P}{\pi} f_s}$$

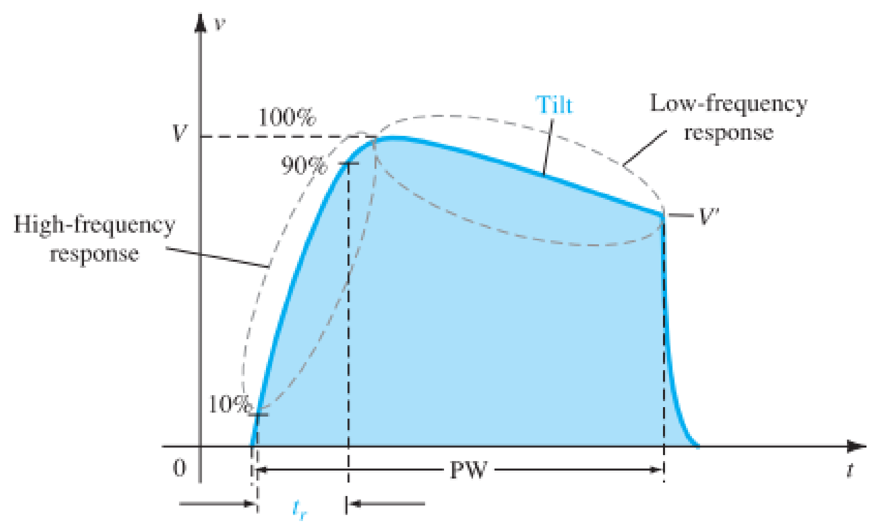


FIG. 9.62
Defining the rise time and tilt of a square wave response.

- For more details, refer to:
  - Chapter 9 at R. Boylestad, **Electronic Devices and Circuit Theory**, 11^{th} edition, Prentice Hall.
- The lecture is available online at:
  - http://bu.edu.eg/staff/ahmad.elbanna-courses/11966
- For inquires, send to:
  - ahmad.elbanna@feng.bu.edu.eg